VOLUME 79 SEPARATE No. 371

PROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

DECEMBER, 1953

DISCUSSION OF PROCEEDINGS - SEPARATES

106, 228

SOIL MECHANICS AND FOUNDATIONS DIVISION

Headquarters of the Society
33 W. 39th St.
New York 18, N. Y.

PRICE $0.50 PER COPY

Current discussion of papers sponsored by the Soil Mechanics and Foundations Division is presented as follows:

This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 22 West Thirty-ninth Street, New York 18, N.Y.

DISCUSSION OF ANALYSIS OF GROUND-WATER LOWERING ADJACENT TO OPEN WATER PROCEEDINGS-SEPARATE 106

MATTHEW I. RORABAUGH,[4] A. M. ASCE.—An orderly analysis of ground-water lowering adjacent to open water is presented in this paper, the entire analysis being based on the assumption that the total discharge is uniformly distributed among an infinite number of wells in a ring or, in other words, that each wellpoint is pumped at the same rate. The theoretical water level within the ring was found to dip away from the river; experimental data were presented indicating a dip toward the river. This difference was explained as probably being due to the pump location and pipe friction. To the writer it seems that the analysis is based on an assumption that does not fit the problem and that the difference between theoretical and observed results is only partly the result of pipe friction.

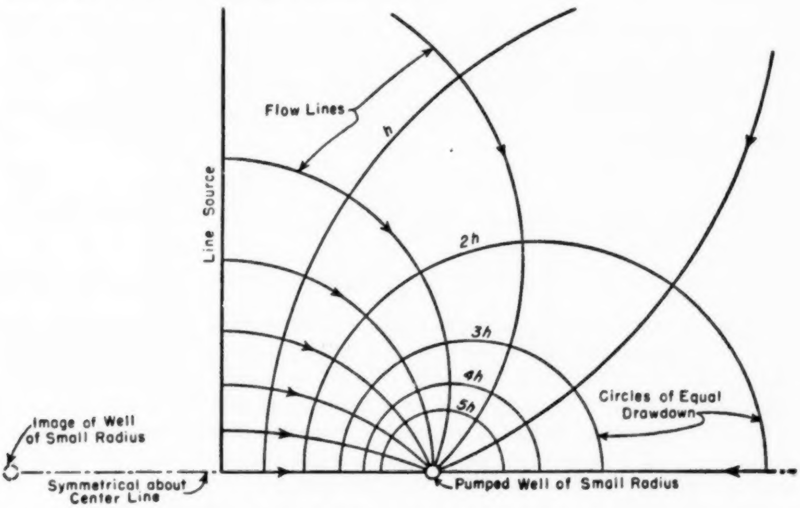

FIG. 15.—FLOW NET FOR PUMPED WELL ADJACENT TO OPEN WATER

In practice, a suction pump is connected to a line or ring of wellpoints. The system is one of constant drawdown at all points on the ring, not uniformly distributed pumping. An analysis based on constant drawdown will be consistent with the field installation and will produce a solution that fits the problem—that is, it is presumed that the excavation is level and that a horizontal, rather than a dipping, water level is desired.

Fig. 15 shows a flow net for the case of a pumping well of very small diameter located near a surface source of water. The flow lines are arcs of circles having their centers on the line source. The lines of equal drawdown are nonconcentric circles having their centers on a line normal to the line source.

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The theory and equations for this system (for the artesian case) have been published by Morris Muskat[5] and C. E. Jacob.[6]

This system is readily adaptable to the problem of a ring pumped at constant drawdown. All that must be done is to place the ring on one of the circles of equal drawdown rather than concentric with the hydraulic center of the system. Pumping from the ring will not be uniformly distributed, being greater on the riverward side where gradients are steeper and stream lines are more dense, and less on the landward side where gradients are less steep and stream lines are less dense. The flow pattern outside the ring will be the same for pumping the ring as for pumping a small well at the same rate, if the small well is located at the hydraulic center.

The drawdown at any point in the field for the case of a small-radius pumped well is expressed by the equation,

$$z^2 = H^2 - \frac{q}{\pi k} \log_e \frac{\sqrt{4\,a^2 + r^2 - 4\,a\,r\cos\theta}}{r} \qquad (22)$$

in which q is the pumping rate; a, the distance from the line source to the center of the well; and r, the distance from the pumped well (or hydraulic center of flow system) to any point at which drawdown is desired. Other terms are as defined in the paper and as shown in Fig. 16. For the riverward profile, $\theta = 0$, and

$$z^2_d = H^2 - \frac{q}{\pi k} \log_e \frac{2\,a - d}{d} \qquad (23)$$

For the landward profile, $\theta = 180°$, and

$$z^2_{d}{}' = H^2 - \frac{q}{\pi k} \log_e \frac{2\,a + d'}{d'} \qquad (24)$$

Eqs. 23 and 24 are similar to Eqs. 11*a* and 11*c* except that the former are for a single small-diameter pumped well located a distance a from the source, whereas the latter are for a ring being pumped at a uniformly distributed discharge, the center of the ring being at a distance p from the source.

The location of the center of the ring with respect to the hydraulic center of the flow net is determined as follows: Let the center of the ring be at a distance p from the source. Then, the eccentricity, e, is $p - a$.

The term e can be evaluated by writing Eqs. 23 and 24 for two points on the specified profiles at locations diametrically opposite each other on a ring of radius r_1; thus, in Eq. 23, $d = r_1 - e$; in Eq. 24, $d' = r_1 + e$. Since the drawdown is the same at all points on the ring, the log terms of these two equations are equal. From this relationship,

$$e = p - \sqrt{p^2 - r^2_1} \qquad (25)$$

Since $e = p - a$, then

$$a = \sqrt{p^2 - r^2_1} \qquad (26)$$

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[5] "The Flow of Homogeneous Fluids Through Porous Media," by Morris Muskat, McGraw-Hill Book Co., Inc., New York, N. Y., 1937 p. 175.

[6] "Engineering Hydraulics," Ed. by Hunter Rouse ("Flow of Ground Water," by C. E. Jacob, Chapter 5), John Wiley & Sons., Inc., New York, N. Y., 1950, p. 345.

Thus, if the location of the center of the ring, p, and the radius of the ring, r_1, are known, the location of the hydraulic center a is determined. Knowing a, the profiles on the riverward and landward sides are computed from Eqs. 23 and 24.

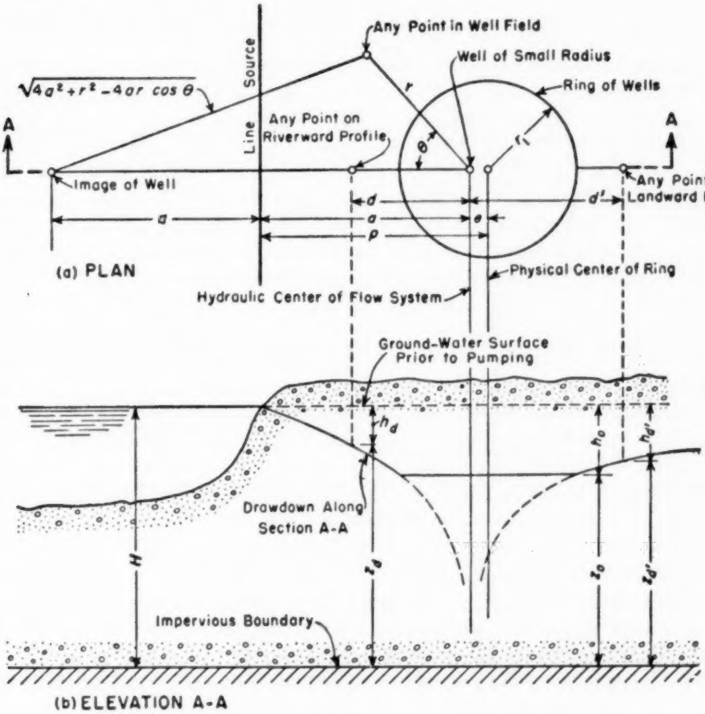

FIG. 16.—RING OF WELLS ADJACENT TO OPEN WATER

The equation for discharge for a given drawdown at any point on the ring or any point within the ring is determined from Eq. 23 or Eq. 24 by substituting $d = r_1 - e$; $d' = r_1 + e$; $e = p - \sqrt{p^2 - r^2_1}$; $a = \sqrt{p^2 - r^2_1}$; $z_d = z_d' = z_o$; $h_d = h_d' = h_o$; and $z_o = H - h_o$; thus—

$$q = \frac{\pi k (2 H h_o - h^2_o)}{\log \dfrac{p + \sqrt{p^2 - r^2_1}}{r_1}} \qquad (27)$$

Eq. 27 is similar to Eq. 6 except that eccentricity is included. For a single well or a very small ring, when r_1 is small relative to p, the log term approaches $\frac{2p}{r_1}$ as in Eq. 6.

The significance of the constant drawdown approach is demonstrated in Fig. 17, which shows profile A, Fig. 11, and the solution for the same case by the constant drawdown method. Note that the constant drawdown solution produces a horizontal profile within the ring which is consistent with the results desired and is consistent with the field condition of a group of wellpoints connected to the same pump. It should be emphasized that, for this partic-

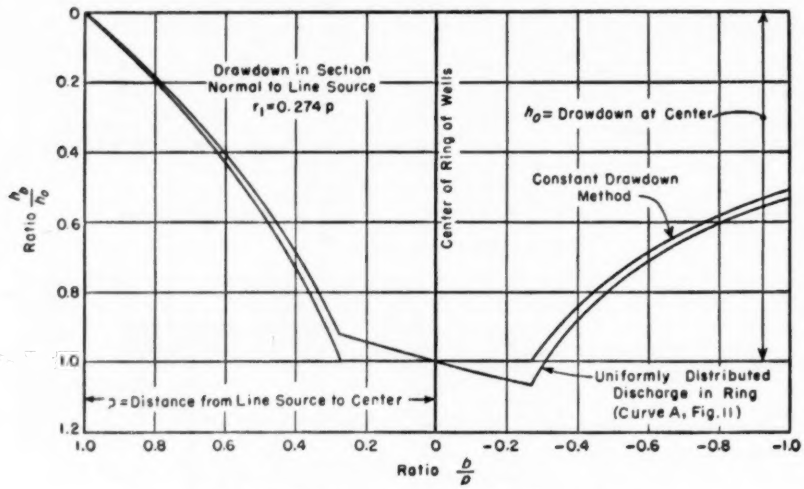

FIG. 17.—DRAWDOWN CURVES BY TWO METHODS

ular case, the eccentricity is small, so that the discharge required to produce the same center drawdown, as determined by Eq. 27, is only about 1.0% larger than that determined from Eq. 6.

The discharge as computed by the two methods differs only in the log term. The discharge required to produce the same center drawdown for the constant drawdown method will be larger than that by the uniformly distributed discharge method by the ratio $\dfrac{\log \dfrac{2\,p}{r_1}}{\log \dfrac{p + \sqrt{p^2 - r^2{}_1}}{r_1}}$. As r_1 approaches p (that is, for excavations very close to the surface source) this ratio will become large; for $p = r$, the ring becomes tangent to the source, and both methods break down since this condition would infer direct pumping from the surface source.

The differences in the profiles outside the ring will become larger as r_1 approaches p, as shown by comparing Eqs. 11*a* and 11*c* with Eqs. 23 and 24. For the constant drawdown method, as the ratio r_1/p is made larger the eccentricity is increased, a becomes smaller (the hydraulic center moves riverward), and the riverward profile becomes steeper. The profiles outside the ring based on Eqs. 11*a* and 11*c* are independent of r_1 so that changing r_1 does not alter the profiles.

This discussion has been limited to the problem of a single ring; but the criticism of the basic assumption of a ring having equal distribution of discharge rather than a ring of constant drawdown applies to all the variations covered in the paper.

Two other points might be mentioned briefly: (1) The location of the line source and (2) the effects of river-water temperature.

1. *Location of the Line Source.*—In problems of water-supply development based on induced infiltration of river water at locations along the Ohio River, it has been determined that the line source is not at the shore line but is usually several hundred feet offshore. The theory assumes a vertical boundary between the source and the aquifer. In nature, water is entering the aquifer vertically in the river bed and then flowing horizontally toward the well. The problem is handled by replacing the complicated flow conditions under the river by an "effective" line source located at a position off shore so that the flow pattern in the well field is approximated. In the case of water-supply development, location of the "effective" line source is determined by controlled field pumping tests, inasmuch as its location may be important in establishing the minimum yield of the installation. Assumption of the shore line as the line source in problems of unwatering is on the safe side; that is, the actual pumping rate should prove to be less than computed.

Effects of River-Water Temperature.—The author uses the definition: "k is the effective coefficient of permeability." Normally ground-water temperature is nearly constant and the viscosity does not vary greatly, so that small errors are lumped into the permeability term. The viscosity of water in the range encountered in the problem in question varies about 1.5% per degree Fahrenheit; thus a rise of 1° F will increase flow rates about 1.5%. The temperature of the surface source becomes an important item in the problem if unwatering is continued over a long time. As an example of the range involved, Ohio River water varies in temperature from 32° to about 85° during the year. Continued pumping near the river induces water of varying temperature to enter the aquifer and flow to the point of pumping. The problem becomes complicated as river water mixes with ground water of a different temperature, heat exchange occurs between the water and sand, and flow rates at each point adjust themselves to the temperature prevailing at that point. The distance from the river is an important item. Detailed records at an installation at Louisville, Ky., for the period from 1945 to 1951 show a range in temperature from 47° to 64° for the discharged water. Statistical analysis shows the flow rate for a condition of constant river level and constant pumping level to vary from 85% to 115% of the average value. In the case of unwatering over a long period, the pumping rate might vary as much as 30% or 40% between summer and winter. Also, this factor will cause variations in the pumping distribution in the ring—in the summer, discharge from wellpoints on the riverward side will increase and in the winter, discharge on the riverward side will decrease. Perhaps Mr. Avery can furnish information as to how long pumping continued in his example, what seasons it covered, whether temperatures of discharged water were recorded, and

whether any differences in pumping rate or water level were noted which would be related to the temperature variable.

Inasmuch as the temperature variable was neglected in the analysis, the six assumptions given at the beginning of the paper should be increased to seven by adding (*g*) The river temperature and groundwater temperature are equal and do not change during the pumping period.

The conclusion that errors as great as 10% to 15% are to be expected might be questioned. Where pumping tests have been run to evaluate the "effective" distance to the line source and the permeability, and where temperature has been included in the computations, water supplies based on induced infiltration generally have been predicted with this degree of accuracy. However, if the line source is assumed as the shore line and the temperature is neglected, greater errors should be expected.

Summary.—On the basis of the assumptions made, the paper presents a sound, orderly treatment of the problem.

The assumption that discharge is uniformly distributed in the ring does not fit the problem, inasmuch as a sloping profile is produced by the theory whereas a horizontal profile is desired. Analysis based on the assumption of constant drawdown at the ring will produce the desired horizontal profile and will be consistent with the field operation of pumping at constant drawdown.

Observed results confirm the view that the constant drawdown approach gives a closer check between theory and practice.

Effects of river temperature were not considered in the analysis. Neglecting this item may introduce errors of as much as 30% or 40%. This item alone is considerably larger than the 10% to 15% limit of error assumed by the author.

S. J. Johnson,[7] A. M. ASCE.—The dearth of information on the engineering aspects of ground-water lowering for construction purposes and the need for such information combine to make this paper one of unusual timeliness and importance. In the United States, ground-water lowering for construction purposes has been all too often resorted to as an expediency when in trouble or used without recognition of its possibilities and limitations. The practical aspects of ground-water lowering operations are of primary importance and an impressive quantity of working knowledge is available. However, the theoretical aspects have not been developed simultaneously with the practical aspects, even though theoretical considerations can supplement field experience to advantage and can also form a logical framework for the acquisition and organization of practical knowledge and experience. Publication of this paper, reviewing some of the available theoretical methods, is therefore of significant value. It is unfortunate that the scope could not have been extended to include a complete review of ground-water lowering theory.

It would have been of some interest and importance if the author had reviewed the status of the available knowledge on ground-water lowering methods

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of analysis. The writer's understanding of this subject is as follows: The equation for gravity flow to a single well, which depends on the validity of the Darcy law,[8] was formulated by Mr. Dupuit in 1863,[3] as was stated by the author. Before 1900, Mr. Forchheimer developed the equations for flow to groups of wells and used the method of images to develop equations for the flow to wells supplied by a line source.[9,10] The formulas for practically all cases of the flow of water to either well or wellpoint systems (either single or multiple stage with the wells arranged in a line, rectangle, or circle) were presented prior to 1930. For example, the first edition of the book by W. Kyrieleis[11] on ground-water lowering by pumping from wells was published in 1913 and presented most of the equations needed for calculating the effect of pumping from wells. The second edition, by Mr. Kyrieleis and W. Siehardt, was published in 1930. Other references are those by H. Weber[12] and Mr. Siehardt[13]; the latter contains formulas for multiple-stage well systems supplied by a circular source. Unfortunately, practically none of the available theory is found in American engineering literature. Additional investigations are needed to develop further certain theoretical considerations, but the available theory is sufficient for many practical applications. Therefore, the significance of this paper is readily apparent.

The Dupuit assumptions have been subjected to much criticism and many have questioned the usefulness of the theoretical methods for estimating drawdown which are based upon it, although its usefulness for satisfactorily estimating the quantity of flow has been acknowledged. Mr. Casagrande has pointed out that the Dupuit assumptions are satisfactory if the slope of the lowered water surface is not too great. This has been confirmed through model tests by H. E. Babbitt, M. ASCE, and D. H. Caldwell,[14] A. M. ASCE, in relaxation solutions by Shih-Te Yang,[15] and by the model tests and relaxation studies of H. P. Hall.[16] Although extensive data are not available, it appears that the Dupuit assumptions do not involve an excessive error in the estimated drawdown at a distance beyond from ten to fifteen well diameters from the center of the well. Consequently, it can be assumed that the Dupuit-Forchheimer theory for steady-state gravity flow to well systems is valid for ordinary requirements in estimating the quantity of flow and the lowered ground-water level, except that the computed values of the lowered ground-water level near

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[8] "Les fontaines publiques de la Ville de Dijon," by H. Darcy, Dijon, France, 1856.

[9] "Über die Ergiebigkeit von Brunnenanlagen und Sickerschlitzen," by Ph. Forchheimer, *Zeitschrift des Architekten und Ingenieuren Vereins zu Hannover*, Vol. 32, 1886.

[10] "Grundwasserspiegel bei Brunnenanlagen," by Ph. Forchheimer, *Zeitschrift des oesterrlichischen Ingenieur und Architekten-Vereins*, Vol. 50, 1898.

[11] "Grundwasserabsenkung bei Fundierungsarbeiten," by W. Kyrieleis, Julius Springer, Berlin, Germany, 1928.

[12] "Die Reichweite von Grundwasserabsenkungen mittles Rohrbrunnen," by H. Weber, Julius Springer, Berlin, Germany, 1928.

[13] "Das Fassungsvermogen von Rohrbrunnen und seine Bedeutung für die Grundwasserabsenkung insbesondere für grossere Absenkungstiefen," by W. Siehardt, Julius Springer, Berlin, Germany, 1928.

[14] "The Free Surface Around, and Interference Between, Gravity Wells," by H. E. Babbitt and D. H. Caldwell, *Bulletin No. 30*, Univ. of Illinois, Urbana, Ill., Jan 7, 1948.

[15] "Seepage Toward a Well Analyzed by the Relaxation Method," by Shih-Te Yang, thesis presented to Harvard University, in Cambridge, Mass., in 1949, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[16] "Investigation of Steady Flow Towards a Gravity Well," by H. P. Hall, thesis presented to Harvard University, in Cambridge, Mass., in 1950, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

the wells will be considerably too low. However, this does not affect the practical usefulness of the theory, since it is the lowering at a distance from the wells—as at the center of an excavation—that is the practical interest.

The author's procedure in developing Eq. 3 is not clear and the resulting effect is that his mathematics appear to be incorrect. Apparently, this is because of his desire to conserve space, but the reader cannot readily supply the missing steps. The author assumes for simplicity that there are an infinite number of wells in Eq. 3 and loses the reader in proceeding to Eq. 4. For equal well discharges, Eq. 1 becomes:

$$H^2 - z^2_o = \frac{q}{\pi\, k}\left[\sum_{i=1}^{i=n} \log_e S_i - \sum_{i=1}^{i=n} \log_e R_i\right] \qquad (28)$$

in which $i = 1, 2, \cdots n$. For wells of diameter $r_1\, d\theta$ so close that they touch, the number of wells, n, equals $2\,\pi\,\frac{r_1}{r_1\, d\theta}$ or $n = \frac{2\,\pi}{d\theta}$ so that the flow per well becomes $\frac{Q\, d\theta}{2\,\pi}$, in which Q is the total flow from the system. Thus, introducing the expression for $S_i = \sqrt{4\, p^2 + r^2{}_1 - 4\, p\, r_1 \cos\theta}$, Eq. 28 becomes

$$H^2 - z^2_o = \frac{Q}{\pi\, k} \times \left[\frac{d\theta}{2\,\pi}\sum \log_e S_i - \frac{d\theta}{\pi}\sum \log_e R_i\right] \qquad (29)$$

An evaluation of $\sum \log_e S_i\, d\theta$ is the integration referred to in Eq. 4, which results in Eq. 5.

The writer has found it convenient to express the terms in the brackets in Eq. 28 in the forms of dimensionless expressions and graphs for a finite number of wells. If Q is the total well flow, Eq. 28 may be expressed as

$$H^2 - z^2_o = \frac{Q}{n\,\pi\, k}\left[\sum_{i=1}^{i=n} \log_e \frac{S_i}{R_i}\right] \qquad (30)$$

which becomes, for a circular well arrangement, since $\sum_{i=1}^{i=n} \log_e R_i = n \log_e r_1$

$$H^2 - z^2_o = \frac{Q}{\pi\, k}\left[\frac{1}{n}\sum_{i=1}^{i=n} \log_e S_i - \log_e r_1\right] \qquad (31)$$

Using the notations from Fig. 4,

$$\left.\begin{aligned} S_i &= \left[r^2{}_1 + (2\, p)^2 - 4\, r_1\, p \cos\,(i-1)\,\frac{2\,\pi}{n}\right]^{\frac{1}{2}} \\ \text{or} \quad & \\ S_i &= r_1\left[1 + 4\left(\frac{p}{r_1}\right)^2 - 4\left(\frac{p}{r_1}\right)\cos\,(i-1)\,\frac{2\,\pi}{n}\right]^{\frac{1}{2}} \end{aligned}\right\} \qquad (32)$$

and

$$\log_e S_i = \log_e r_1 + \frac{1}{2}\log_e\left[1 + 4\left(\frac{p}{r_1}\right)^2 - 4\left(\frac{p}{r_1}\right)\cos\,(i-1)\,\frac{2\,\pi}{n}\right] \qquad (33)$$

The summation $\frac{1}{n}\sum_{i=1}^{i=n} \log_e S_i$ now can be expressed in dimensionless form and plots of it can therefore be simply prepared. Eq. 31 thus becomes

$$\left.\begin{aligned} H^2 - z^2_o &= \frac{Q}{\pi k}\left[\log_e r_1 + L - \log_e r_1\right] \\ \text{or} \qquad H^2 - z^2_o &= \frac{Q\,L}{\pi k} \end{aligned}\right\} \quad (34)$$

where

$$L = \frac{1}{2n}\sum_{i=1}^{i=n} \log_e\left[1 + 4\left(\frac{p}{r_1}\right)^2 - 4\frac{p}{r_1}\cos\,(i-1)\,\frac{2\pi}{n}\right] \quad (35)$$

The value of L is plotted in Fig. 18 for various values of p/r_1 and different numbers of wells. The flow per well is assumed to be the same for all wells. It can be seen that L is only slightly dependent on the number of wells used. Therefore, the expression, $L = \log_e 2\,p/r_1$, as given (in effect) by the author in Eq. 5, may be used regardless of the number of wells when p/r_1 is greater than about 2, and for a moderate number of wells when p/r_1 is less than 2.

The equation for the lowered water surface in the wells, when a limited number of wells is used, can be obtained simply and is of interest. If the distance from the impervious layer to the water surface in the well is denoted by z_w, it is apparent from Eq. 1 that for a circular arrangement of wells with an equal flow per well

$$H^2 - z^2_w = \frac{Q}{\pi k}\left[\frac{1}{n}\sum_{i=1}^{i=n} \log_e S_i - \frac{1}{n}\sum_{i=1}^{i=n} \log_e R_i\right] \quad (36)$$

It is apparent that $\frac{1}{n}\sum_{i=1}^{i=n} \log_e S_i$ can be obtained from the previous evaluation of this summation by simply letting $2\,p$ equal the distance from the well at which the drawdown is desired to the center of the imaginary well system, which distance may be called $2\,p'$. For most well systems the value of the summation can be approximated by $\log_e r_1 + \log_e \frac{2\,p'}{r_1}$. Mr. Sichardt explains that

$$\frac{1}{n}\sum_{i=1}^{i=n} \log_e R_i = \log_e r_1 - \frac{1}{n}\log_e \frac{r_1}{n\,r_w} \quad (37)$$

in which r_w is the well radius.[13] It follows that the evaluation of the water in the well may be found from the equation,

$$H_2 - z^2_w = \frac{Q}{\pi k}\left[\log_e \frac{2\,p'}{r_1} + \frac{1}{n}\log_e \frac{r_1}{n\,r_w}\right] \quad (38)$$

The author limits his discussion to circular well arrangements. If a rectangular system of wells or wellpoints is used, it may be converted into an equivalent circular system having an enclosed area equal to the area of the

rectangle, or the rectangular system can be analyzed by using the graph prepared by Mr. Weber.[12] The writer has found it convenient also to prepare dimensionless graphs of $\sum \log_e S_i$ and $\sum \log_e R_i$ for lines of wells. These graphs also can be used to find the lowering at any point when using a rectangular area.

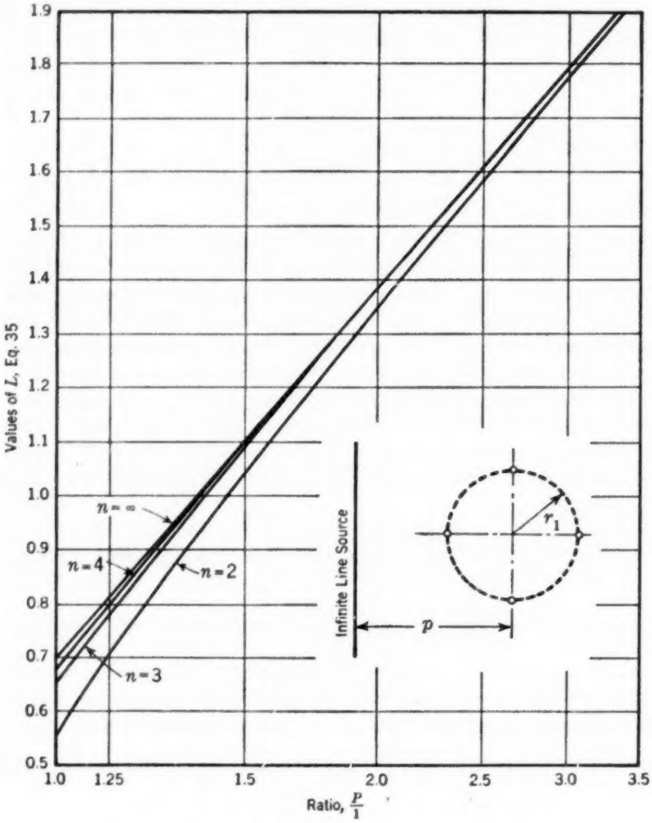

FIG. 18.—CIRCULAR SYSTEM WITH INFINITE LINE SOURCE

The example given by the author is particularly interesting because it demonstrates an engineering approach to a ground-water lowering operation. Unfortunately, there are only a limited number of cases where such an approach is actually used. The author did not indicate how he treated the fact that the wells only partly penetrate the previous stratum. This is a practical feature of considerable importance. He apparently assumed that the wells completely penetrated the pervious stratum, which is a different assumption from that sometimes made. The author's viewpoint on this phase would be appreciated. The length and spacing of the wellpoints are pertinent and should, perhaps, be indicated.

In conclusion, the writer wishes to emphasize his belief in the substantial significance of this paper and in the practicability of applying this approach. At present, only a few engineers have been applying in practice the available theory for ground-water lowering. It is hoped that the paper will stimulate interest in this field.

HOWARD P. HALL,[17] A. M. ASCE.—Convenient expressions are developed in this paper for determining the drawdown surface in a number of multiple-well problems that have not been treated in such detail before. The writer would like to outline an analytical approach that may simplify the derivation of the equations to some extent, and thereby facilitate further investigation.

Fig. 19 illustrates the case of a single gravity well near open water in the form in which it was originally analyzed by Mr. Forchheimer.[18] The ground-water surface before the beginning of operation of the well is assumed to have been horizontal at the elevation of the open water surface, the pervious stratum is assumed to be homogeneous and isotropic, and the well is assumed to penetrate the full depth of pervious material to an underlying horizontal

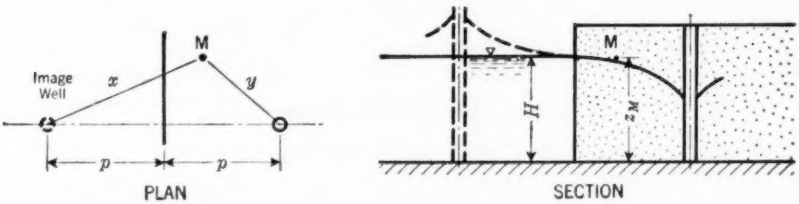

FIG. 19.—THE FORCHHEIMER SOLUTION APPLIED TO A SINGLE WELL NEAR A RIVER

impervious base. Mr. Forchheimer imagined the body of open water to be replaced by a mirror image of the given well and the semi-infinite soil mass, and required that the imaginary well feed water into the pervious stratum at the same rate at which it was being drawn out of it through the real well. Conditions of flow in the given soil mass would not be affected by this substitution, but the problem could now be treated as a very simple well group. Making use of an analysis which he had already developed for a group of wells,[19] Mr. Forchheimer derived the equation:

$$z^2 = H^2 - \frac{q}{\pi k} \log_e \frac{x}{y} \qquad (39)$$

in which z is the elevation of an arbitrary point on the free surface, measured from the impervious base; H is the elevation of original ground-water surface, measured from the impervious base; q is the rate of flow from the imaginary well to the given well; k is the coefficient of permeability of the pervious stratum; and x, y are the distances from the arbitrary point to the imaginary well and given well, respectively.

Referring now to the problem in the paper, let Fig. 20 represent a typical arrangement of a number of wells in a circle having a radius r_1 and a center at a distance p from open water. Eq. 4 and Eq. 11 have been derived with

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[18] "Grundwasserspiegel bei Brunnenanlagen," by Ph. Forchheimer, *Zeitschrift des oesterreichischen Ingenieur und Architekten-Vereins*, Vol. 50, 1898, pp. 629–648.

[19] "Über die Ergiebigkeit von Brunnenanlagen und Sicherschlitzen," by Ph. Forchheimer, *Zeitschrift des Architekten und Ingenieuren Vereins zu Hannover*, Vol. 32, 1886, pp. 539–564.

such an arrangement in mind, but with the well group idealized as a ring composed of an infinite number of small wells—that is, as a well having an annular cross section. Examination of the form of these equations suggests that the problem might be analyzed by replacing the given ring of wells with

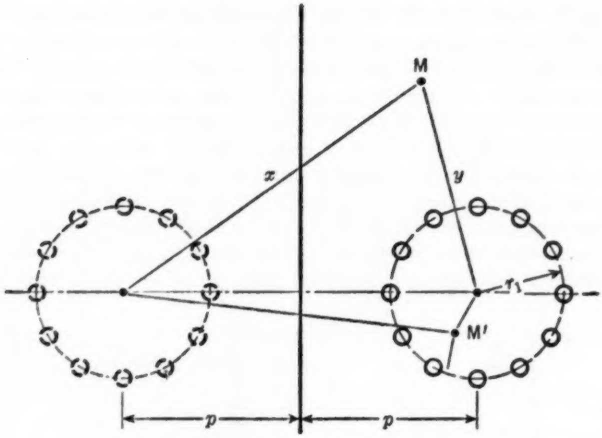

FIG. 20.—RING OF WELLS AND AN EQUIVALENT SINGLE WELL

a single well having the same center and radius as the ring, and discharging at a rate equal to the sum of the rates of discharge of the given wells. Thus, if n is the number of given wells and q is the rate of discharge from each well, Eq. 39, applied to this substitute well, becomes—

$$z^2_M = H^2 - \frac{n\,q}{\pi\,k}\log_e\frac{x}{y} \qquad (40)$$

—for any point, M, outside the ring.

Within the ring, Eq. 39 may also be used, but it must be noted that, theoretically, the drawdown contours within this area are concentric circular arcs whose center is the center of the image ring. (The original analysis superposed the drawdown of the given well upon the negative drawdown of the image well.) Thus, the elevation of the drawdown surface at any point within the ring is the same as that of a point on the ring circumference at the same distance from the image center. Eq. 39 for a point, M′ (Fig. 20) within the ring may therefore be written:

$$z^2_{M'} = H^2 - \frac{n\,q}{\pi\,k}\log_e\frac{x}{r_1} \qquad (41)$$

A comparison of Eqs. 40 and 41 with Eqs. 11 shows that they are the same. It should be noted that the introduction of the idea of an equivalent single well does not reduce the time and labor required for computing numerical results since the equations are the same, but the derivation is somewhat simpler since the analytical procedure is reduced to the direct application of one equation.

The foregoing discussion indicates only that the equivalent single well may be substituted for a well of annular cross section without loss of theoretical

accuracy. It is also of importance to know to what extent the number of wells can be reduced and the radius of ring increased in relation to p (Fig. 20) before the device loses its theoretical validity. Fig. 11 contains drawdown curves for a ring of six wells, which can be compared to the curve for the idealized annular well, and, therefore, to the equivalent single well, under similar conditions. The differences indicated are not large. The following considerations extend this comparison a little further.

Consider a group of n equal wells, each discharging at a rate q, located on a circle whose radius is r_1 and whose center is at a distance p from the shore line, as in Fig. 20. Let r_e be the radius of a single well having the same center as the given ring and the same total rate of discharge as the n given wells, and producing the same drawdown at the center; that is, let r_e be the radius of the equivalent single well corresponding to a ring composed of a finite number of wells.

Beginning with the direct analysis, the following is obtained from Eq. 1:

$$z^2_o = H^2 - \frac{q}{\pi k} \sum_1^n \log_e \frac{x_n}{r_1} \qquad (42)$$

in which z_o is the elevation of the free surface at the center of the ring, and x_n is the distance from the center of the ring to the image of the nth well.

An analysis of the same case by the equivalent single well gives

$$z^2_o = H^2 - \frac{n\,q}{\pi k} \log_e \frac{2\,p}{r_e} \qquad (43)$$

If the drawdown at the center is to be the same, it follows that

$$n \log_e \frac{2\,p}{r_e} = \sum_1^n \log_e \frac{x_n}{r_1} \qquad (44)$$

or

$$\left(\frac{2\,p}{r_e}\right)^n = \frac{x_1, x_2, \cdots, x_n}{(r_1)^n} \qquad (45)$$

and

$$\frac{r_e}{r_1} = \frac{2\,p}{\sqrt[n]{x_1, x_2, \cdots, x_n}} \qquad (46)$$

Eq. 46 is a convenient form in which to compare the radius of the equivalent single well to the radius of the given ring of wells. If p is large in relation to r_1, it follows that $x_1 \approx x_2 \approx \cdots \approx x_n \approx 2\,p$, and the ratio reduces to unity, as would be expected. As r_1 approaches the magnitude of p, the corresponding changes in the ratio are insignificant until the circumference of the ring of wells comes very close to the shore line. For example, in the case of a ring of as few as four wells located at the quarter points, the radius of the equivalent single well is less than 1% different from the ring radius until p becomes less than $1.125\,r_1$. Consequently, it appears that, from the point of view of drawdown at the center, the equivalent single well is an acceptable analytical device in most cases.

The differences at the circumference of the ring are the greatest, as Fig. 11 indicates. Eq. 1 applied to a point on the circumference of the ring may be written

$$H^2 - z^2 = \frac{q}{\pi k} \sum_1^n \log_e \frac{x_n}{y_n} \qquad (47a)$$

The corresponding expression using the equivalent single well becomes

$$H^2 - z^2 = \frac{n\,q}{\pi\,k} \log_e \frac{x}{r_1} \qquad (47b)$$

In cases where the drawdown is assumed to be not greater than 0.2 H, Mr. Casagrande recommends the following approximation as introducing an error of not more than 10%:

$$H^2 - z^2 = (H + z)\,(H - z) \approx 2\,H\,h \qquad (47c)$$

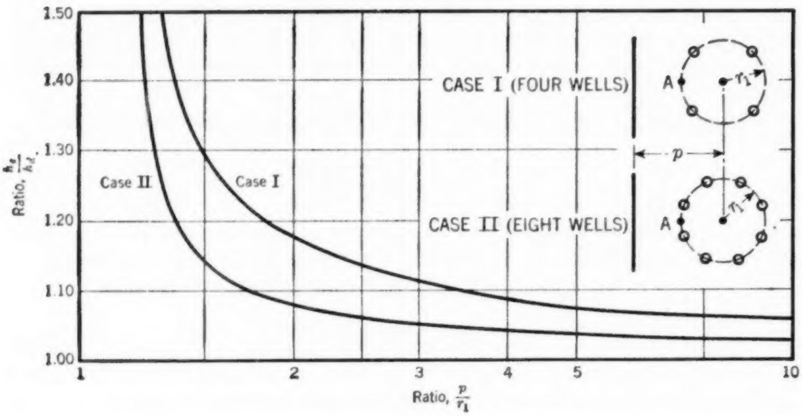

FIG. 21.—RATIO $\frac{p}{r_1}$ VERSUS $\frac{h'_e}{h_d}$ AT POINT A

in which h is the drawdown $= H - z$. If Eq. 47b is divided by Eq. 47a with this approximation introduced, the resulting equation expresses the ratio of drawdowns computed by the two methods:

$$\frac{h_e}{h_d} = \frac{\log_e \left(\frac{x}{r_1}\right)^n}{\log_e \frac{x_1,\,x_2,\,\cdots,\,x_n}{y_1,\,y_2,\,\cdots,\,y_n}} \qquad (48)$$

in which h_e is the drawdown determined by use of the equivalent single well and h_d is the drawdown determined by summation of the contributions of individual wells.

Eq. 48 provides the basis for the curves of Fig. 21. A ring of four wells and one of eight are considered separately. Each group is arranged so that the point on the ring circumference which is nearest the shore line is also midway between two wells. Thus, point A corresponds to the largest discrepancy between the equivalent single well solution and the conventional summation for a given number and location of wells. In view of the fact that these differences are localized, as the six-well drawdown curve of Fig. 11 indicates, it appears from Fig. 21 that, even at the ring circumference, the equivalent single well is reasonably reliable in most cases.

DISCUSSION OF REDUCTION IN SOIL STRENGTH
WITH INCREASE IN DENSITY
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E. S. Barber,[2] A. M. ASCE.—It is reasonable to expect pore pressures for compaction beyond the line of optimums since the phenomenon of an optimum moisture is caused by pore pressures developed in excess water which cannot move away quickly enough. Conversely, no optimum is obtained for very permeable soils or indeed for any soil under static loading if sufficient opportunity for drainage is provided.

Figure A corroborates the reduction in strength from overcompaction by impact; however, the continued increase in strength with density for static compaction indicates that something besides pore pressure is involved. Since shearing is restricted in static compaction, it is suggested that the shearing involved in impact may cause reduced strengths.

To eliminate pore pressure as a factor, tests were made on dry sand. For a fine sand, with a 10-lb. hammer dropping 18 inches, a CBR of 5 as compacted was obtained while compaction to the same density by vibration gave a CBR of 18. Similar tests on a coarse sand gave CBR's of 6 and 16, respectively. Apparently, the reduced density along the shear surfaces from impact is a major factor in reducing strength.

A clayey sand and gravel, initially compacted by impact to 129 p.c.f. with 8.8 percent water was subjected to 100,000 repetitions of 4 kips per sq. ft. producing a density of 120 p.c.f. and a CBR of 47. Repetition of the test with a 2 percent sodium sulfate added to the soil produced a density of 141 and increased the CBR to 132. A similar material with a constant moisture content of 5.7 percent had a soaked CBR of 90 after 25 blows of the 10-lb. hammer; after 55 blows the CBR was reduced to 75. However, with 55 blows plus 7000 repetitions of static load, the CBR was increased to 145.

Thus, the strength may increase or decrease with increased density depending upon the structure produced by the method of compaction.

2. Highway Engr., Physical Research Branch Bureau of Public Roads, U.S. Dept. of Commerce, Washington, D.C.

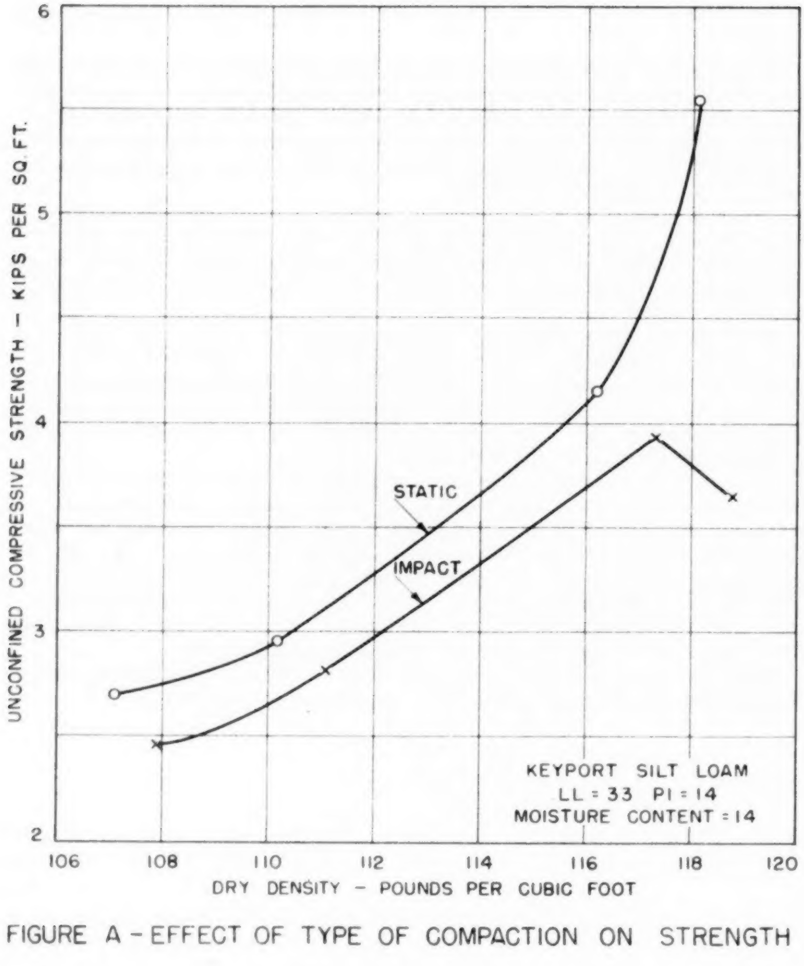

FIGURE A - EFFECT OF TYPE OF COMPACTION ON STRENGTH

George R. Halton,[1] M. ASCE.—The author's theme is well supported by reliable data of the particular type of soil tested. The precautions taken to eliminate contributory variables, as exemplified by the use of new material for each specimen, are commendable. In many respects the author's findings parallel those of an investigation made in 1943 by the California Division of Highways at the Oxnard Flight Strip in Ventura County, California. A few facts and observations of the latter investigation, which was conducted under the writer's direction, are given below.

It will be noted that the Oxnard base material was much coarser than the soil from which the author's data were derived and that the laboratory CBR values at densities equivalent to field conditions at Oxnard did not corroborate the field performance of the relatively coarse material which failed in spots. Otherwise the author's premise of an inverse strength-density relation above certain moisture contents is substantiated by the California study. Both the Vicksburg and the Oxnard data show perched saturation and freedom from the influence of a high water table.

The Oxnard Flight Strip was one of many built during the first years of World War II under the general supervision of the Public Roads Administration with detailed investigation and inspection by state highway departments. The material used in its base and subbase was at that time classified as an A-2 friable to A-2 plastic gravelly sand. It approximated an SC-GF in contrast to the ML-CL soil at Vicksburg.

The native subgrade at the Oxnard site was a silty "adobe" clay estimated to require a cover of 2.5 to 3.0 feet of 15 CBR subbase and 30 CBR base material. A buried bank of gravelly sand, discovered two miles from the strip by the late Col. R. J. Allan[2], provided the needed 280,000 cu. yds. of material which complied with the CBR and other requirements of the specifications.

Relative compaction was specified to be not less than 90% based on the California Method[3] and averaged 96+% in 118 control tests. The special tests of the investigation averaged about 102%. The water content was not specified but was fairly well regulated, averaging 6.6% in place at the time of sampling and 8.8% at optimum. The moisture at the time of rolling was probably one or two per cent higher than when sampled. Only 2 of the 118 control tests exceeded their respective optimums.

The above ratio of 2 in 118 is equivalent to the areal proportion of several base failures which developed under the trucks hauling bituminous plant-mixed leveling course and surfacing. All but a very few of the 45 spots which failed had received bituminous surface treatment when distress appeared. The defective areas exhibited typical retrogression from an initial slight springiness to a final pronounced alligator cracking and rutting.

During the special investigation to determine the cause of failure, 20 samples from failure areas and 6 samples from adjacent areas which showed no distress were subjected to tests of field condition and laboratory properties. Most of the samples were taken within a few days after damage occurred; a few were taken within a few minutes after distress was deliberately induced by truck routing.

Unfortunately neither the equipment nor the precedent for Hveem stabilometer tests of untreated aggregate or for field CBR tests were locally available at that time. The majority of the samples taken in the special investigation

1. Consulting Engineer, Newark, N.J.
2. Assoc. M. ASCE; died Feb. 1946; formerly District Materials Engineer, Calif. Div. of Highways.
3. ASTM Procedures for Testing Soils, July 1950, p. 209.

TABLE 2

COMPARISON OF TESTS OF FAILURES AND NON-FAILURES

Property	FAILURES Range	Avg.	Sample CL-36	NON-FAILURES Sample CL-39	Avg.	Range	Remarks (1)
% Pass. 3/4" Sieve	94-99	96	95	94	95	94-96	
% Pass.No.50 Sieve	34-53	44	41	43	44	43-46	
% Pass.No.200 Sieve	18-32	23	20	20	21	20-22	
% Finer than 5 microns	5-9	6	5	5	6	5-7	
% Finer than 1 micron	1-3	2	1	1	2	1-3	
Liquid Limit	14-20	16	14	14	14	14-16	
Plasticity Index	1-6	3	1	1	1	1	
PRA Class.	-	A-2 Pl.	A2 Fr.	A-2 Fr.	A-2 Fr.	-	(1)
Field Condition* Density, p.c.f	129-135	132	131	127	128	122-129	(1) (2) G=2.65±0.01
Water Content %	7.1-9.4	8.4	8.3	8.3	7.0	5.8-7.9	
Saturation %	74-100	90	87	74	69	58-76	(2)
Laboratory Equivalent of Field Condition:							
Density, pcf	127-132	130	130	127	127	122-129	
Water Content %	7.8-8.4	8.2	8.4	8.2	7.4	6.1-8.2	
Water Content after 4-day soak	7.8-8.3	8.0	8.0	8.7	8.9	8.3-9.7	
Gain (+) or Loss(-)	+0.1 to -0.6	-0.2	-0.4	+0.5	+1.5	+0.5 to +2.2	
CBR Before Soaking	86-154	120	154	112	125	21-196	(1)
CBR After Soaking	80-110	96	83	78	65	27-92	(1)
Expansion %	0.0-0.1	0.1	0.1	0.1	0.2	0.1-0.3	
Standard Lab. CBR							
Density	122-129	124	123	123	124	123-126	(1) (3)
Water Content	7.8-8.8	8.1	8.8	8.8	8.5	8.4-8.8	
CBR	50-87	68	80	69	72	65-85	(3) (1)

(1) In the California Division of Highways in 1943, the Casagrande classification, the impact compaction of CBR specimens and the adjustment of CBR penetration were not standard procedure.
(2) One failure sample with apparent density of 140 and saturation of 127% was outlawed.
(3) ASTM Procedures for Testing Soils, July 1950, Page 209.

were tested in the laboratory for CBR at the approximate field condition as well as at the then standard soaked and unsoaked density achieved under a static load of 2000 p.s.i. The results of the special tests are summarized in Table 2.

Infiltration from 0.8 inch of precipitation in the week preceding failure was an early cause suspect but was deemed immaterial after field permeability tests of the bituminous treated upper 3 inches of base material showed only 2×10^{-4} feet per day. Ground water elevation and capillary action were ruled out as causes of failure when the special tests of natural subgrade material showed it to be relatively dry, dense and stable.

During the repair of the failures the characteristic springiness was observed in only the upper half of the 2.5 to 3.0- foot combined thickness of base and subbase materials. Some of the incipient failures which were not immediately repaired subsequently showed no distress when loaded trucks were deliberately routed over them. Elsewhere one or two spot failures were artificially induced by deliberate repetition of truck passes. Following the interruption due to repairs the paving was completed without the recurrence of old failures or the development of new ones. The paved strip was still in good condition in 1947, beyond which date the writer has no information.

The investigation was hampered somewhat by the late Fall paving tempo of construction and by the immaturity and divergence of opinions which characterize problems lacking in precedent. The disparity between field behavior and routine or standard CBR values did prompt special tests of laboratory CBR at field conditions and in a final field test 3 or 4 weeks after the epidemic of spot failures the rear tire of a loaded truck was driven within 8 inches of a 6-inch diameter auger hole filled with water in a rubber bag.[4] The latter test produced no observable change in water level.

The field and laboratory investigations at Oxnard were made by experienced, expert personnel. Samples from 6-inch diameter auger holes were weighed in the field to the nearest 0.01 pound, placed in metal containers with sealed covers and checked for weight in the laboratory. Calibrated sand backfill from moisture-proof containers was also weighed to 0.01 pound in the field and special depth-density calibrations and techniques assured accurate volume determinations at all horizons in the sample holes. At least 50 pounds of additional soil immediately adjacent to the hole was secured from each horizon sampled, to provide new material for each laboratory specimen. The entire auger portion of each sample was dried at a controlled low temperature and the test specimens were moistened for 24 hours before the final addition of water required to either match the field condition or straddle optimum.

It was concluded in 1943 that the Oxnard failures were due to excessive saturation in the very dense upper half of base and subbase material and that they would have healed or ceased retrogression if left unrepaired for a few weeks. The slightly greater plasticity of the samples from failed areas was not considered a general cause of failure because 93 of 95 Plasticity Index determinations ranged between 0 and 4 because the correlation within that narrow spread was inconsistent. The high values of laboratory CBR at equivalent field condition were attributed to the slow rate of CBR penetration as compared to the deformation under a rolling truck; the CBR loading rate allowed time for the attenuation of excess pore pressures which the truck roll compounded into rupture.

In retrospect these conclusions still seem sound, but the reported differences in density and saturation of the failures as against the non-failures

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4. ASTM Procedures for Testing Soils, July 1950.

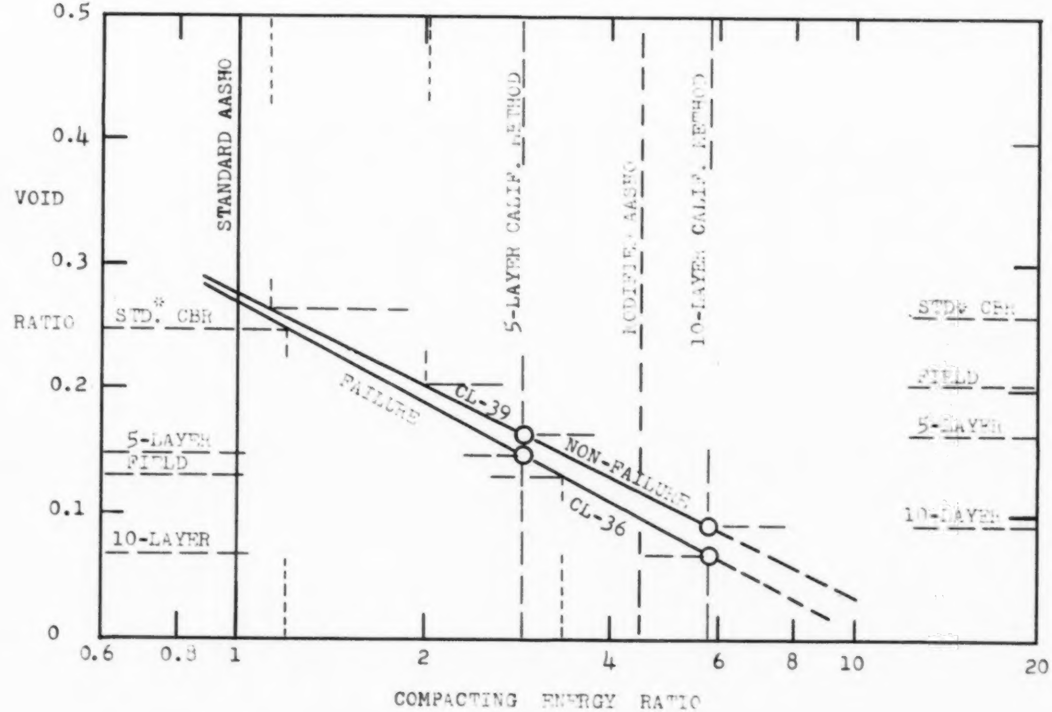
suggest that some unknown factor exaggerated the true differences. The abnormal saturation, the unusual rebound, the difficult laboratory duplication of the reported field densities and the loss of water during the soaking period all suggest that the auger holes in failed areas were subject to bleeding and constriction during the sampling. The inconsistency of computed field densities on an e-log p chart verifies the suspicions suggested above.

The qualitative relation of density to pressure is shown in Fig. 6, wherein the unit on the log p scale is a theoretical energy ratio rather than a numerical quantity of energy per cubic foot (= ft-lbs./ft.^{3} = unit pressure). The virgin slopes were determined from densities achieved in the 5-layer and 10-layer variations of the California Method. The difference in field densities is inconsistent with the uniform, lapped pattern of many roller passes and non-uniform, spotty pattern of actual failures.

The spotty pattern of failed areas suggests that individual loads or groups of loads from drier or wetter portions of the pit may have been an antecedent cause. Delayed absorption in coarse particles wetted shortly before or during rolling probably caused pore pressures, which were "built in" by the roller, to gradually diminish. If the delayed portion of absorption was 0.4% by weight = 1.0% by volume, and rolling left 3.0% by volume of air in the voids between particles, the total air voids would be 4.0%. If the pressure trapped in the 3.0% was 4 tons per square foot, delayed absorption would reduce this toward 3 tons per square foot. Compacted soil which experienced such an internal relief of pore pressure could then withstand an increment of pressure corresponding to 1% of compression.

Soil without delayed particle absorption would lose trapped pressure, but more slowly, via regional diffusion and in proportion with permeabilities and lengths of escape paths. Loads applied before such external relief became effective could withstand only slight increments of pressure without rupture.

From the above considerations in conjunction with the author's data and deductions it seems logical that the critical volume of air voids, although of the order of 3% on the average, is probably a function of load and loading rate as well as a function of the dimensions, type and condition of soil. With behavior subject to so many variables anomalies such as the correlation of field CBR with field strength in one soil and the lack of a similar correlation in another soil are to be expected.



0.5
0.4
VOID
0.3
RATIO
0.2
0.1
0
0.6
0.8
1
2
4
6
8
10
20
COMPACTING ENERGY RATIO
STANDARD AASHO
5-LAYER CALIF. METHOD
MODIFIED AASHO
10-LAYER CALIF. METHOD
STD.* CBR
5-LAYER
FIELD
10-LAYER
FIELD
5-LAYER
CL-39
CL-36
FAILURE
NON-FAILURE

FIGURE 6